

Path-following based Point Matching using Similarity Transformation

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Abstract

To address the problem of 3D point matching where the poses of two point sets are unknown, we adapt a recently proposed path following based method to use similarity transformation instead of the original affine transformation. The reduced number of transformation parameters leads to more constrained and desirable matching results. Experimental results demonstrate better robustness of the proposed method over state-of-the-art methods.

1 Introduction

Point matching is a challenging problem with applications in computer vision and pattern recognition. To solve this problem, the robust point matching (RPM) algorithm [1] uses deterministic annealing for optimization. But it needs regularization to avoid undesirable matching results and has the tendency of aligning the mass centers of two point sets. To address this issue, Lian and Zhang reduced the objective function of RPM to a concave function of the point correspondence variable and used the branch-and-bound (BnB) algorithm for optimization [2, 3]. These methods are more robust, but their worse case time complexity is exponential due to use of BnB. To address this issue, Lian used the path following (PF) strategy [4] to optimize the objective function of [3] by adding a convex quadratic term to the objective function and dynamically changing the weights of the terms [5].

But in the case of 3D matching, the method of [5] is experimentally shown to only perform well when the transformation is regularized, while there are problems where the poses of two point sets are unknown which call for matching methods where the transformations are not

regularized. The reason that the method of [5] performs poorly is that it uses affine transformation which has large number of parameters, thus resulting in high degree of transformation freedom and unconstrained matching results. To address this issue, we modify the method to use similarity transformation whose number of parameters is considerably smaller.

2 The RPM objective function

Suppose we are to match two point sets $\mathcal{X} = \{x_i, 1 \leq i \leq m\}$ and $\mathcal{Y} = \{y_j, 1 \leq j \leq n\}$ in \mathbb{R}^d . For this problem, RPM uses the following mixed linear assignment–least square model:

$$\begin{aligned} \min \quad & \Phi(P, s, R, t) = \sum_{i,j} p_{ij} \|y_j - sRx_i - t\|^2 - \mu 1_m^\top P 1_n \\ & = 1_m^\top P \tilde{y} + s^2 \tilde{x}^\top P 1_n - 2s \cdot \text{tr}(RX^\top PY) + 1_m^\top P 1_n \|t\|^2 \\ & \quad - 2t^\top (Y^\top P^\top 1_m - sRX^\top P 1_n) - \mu 1_m^\top P 1_n \end{aligned} \quad (1)$$

$$s.t. \quad P 1_n \leq 1_m, \quad 1_m^\top P \leq 1_n, \quad P \geq 0, \quad (2)$$

$$\underline{s} \leq s \leq \bar{s} \quad (3)$$

Here we use similarity transformation with R , s and t being rotation matrix, scale change and translation vector. The constants $\underline{s} \geq 0$ and $\bar{s} \geq 0$ are lower and upper bounds of s . The matching matrix $P = \{p_{ij}\}$ has $p_{ij} = 1$ if two points i, j are matched and 0 otherwise. The last term in Φ is used to regularize the number of correct matches with μ being the balancing weight. $\|\cdot\|$ is the l_2 norm of a vector and $\text{tr}()$ denotes the trace of a square matrix. 1_n represents the n -dimensional vector of all ones. The matrices $X \triangleq [x_1, \dots, x_m]^\top$, $Y \triangleq [y_1, \dots, y_n]^\top$ and vectors $\tilde{x} \triangleq [\|x_1\|^2, \dots, \|x_m\|^2]^\top$, $\tilde{y} \triangleq [\|y_1\|^2, \dots, \|y_n\|^2]^\top$.

It's easily seen that given the values of P , s and R , Φ is a convex quadratic function of t . Hence, the optimal t minimizing Φ can be obtained via $\frac{\partial \Phi}{\partial t} = 0$ to be $\hat{t} = \frac{1}{1_m^\top P 1_n} (Y^\top P^\top 1_m - sRX^\top P 1_n)$. Substituting \hat{t} into Φ to eliminate t yields an energy function with reduced number of variables:

$$\begin{aligned} \Phi(P, s, R) &= 1_m^\top P \tilde{y} - \mu 1_m^\top P 1_n - \frac{1}{1_m^\top P 1_n} \|Y^\top P^\top 1_m\|^2 \\ &\quad + s^2 (\tilde{x}^\top P 1_n - \frac{1}{1_m^\top P 1_n} \|X^\top P 1_n\|^2) \\ &\quad - 2s \cdot \text{tr}(R(X^\top PY - \frac{1}{1_m^\top P 1_n} X^\top P 1_n 1_m^\top PY)) \end{aligned} \quad (4)$$

3 Optimal s, R minimizing $\Phi(P, s, R)$

Let matrix

$$A \triangleq X^\top P Y - \frac{1}{1_m^\top P 1_n} X^\top P 1_n 1_m^\top P Y$$

and let USV^\top be the singular value decomposition of A^\top , where S is a diagonal matrix and the columns of U and V are orthogonal unity vectors. Then given $s > 0$, the optimal rotation matrix R minimizing Φ in (4) is $\hat{R} = U \text{diag}([1, \dots, 1, \det(UV^\top)]) V^\top$ [6], where $\text{diag}(\cdot)$ denotes converting a vector into a diagonal matrix and $\det(\cdot)$ is the determinant of a square matrix. Substituting \hat{R} into (4) to eliminate R yields a (possibly concave) quadratic program in single variable s . Given the range of s as $\underline{s} \leq s \leq \bar{s}$, one can easily solve this quadratic program by comparing the function values at the boundary points \underline{s} , \bar{s} and the extreme point.

4 An objective function in one variable P

We aim to obtain an objective function only in one variable P , which can be achieved by minimizing Φ with respect to s and R , i.e.:

$$\Phi(P) \triangleq \min_{s, R} \Phi(P, s, R) \quad (5)$$

For $\Phi(P)$, the following results can be established:

Proposition 1 $\Phi(P)$ is concave under constraints (3).

Proof: Based on the aforementioned derivation, we have $\Phi(P, s, R) = \min_t \Phi(P, s, R, t)$. Consequently, we have

$$\Phi(P) = \min_{s, R} \Phi(P, s, R) = \min_{s, R, t} \Phi(P, s, R, t)$$

For each s , R and t , $\Phi(P, s, R, t)$ is apparently a linear function of P . We see that $\Phi(P)$ is the point-wise minimum of a family of linear functions, and thus is concave, as illustrated in Fig. 1.

The fact that $\Phi(P)$ is concave makes it easier for the PF algorithm to be applied to the minimization of our objective function as it requires two terms, a concave and a convex term, to be provided.

Proposition 2 There exists an integer solution for any local minima (including the global minimum) of function $\Phi(P)$ under constraints (2) and (3).

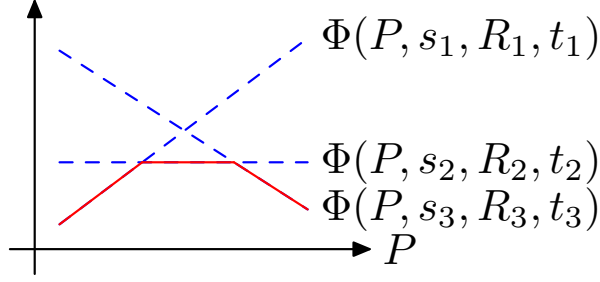


Figure 1: Point-wise minimization of a family of linear functions $\Phi(P, s, R, t)$ (dashed straight lines) with respect to parameters s , R and t results in a concave function (solid piecewise straight line).

Proof: The polytope formed by constraint (2) satisfies the total unimodularity property [7], which means that the coordinates of the vertices of this polytope are integer valued. We already proved that $\Phi(P)$ is concave under constraints (3). It is well known that any local minima (including the global minimum) of a concave function over a polytope can be obtained at one of its vertices. Thus, the proposition follows.

This result implies that minimization of $\Phi(P)$ by simplex-like algorithms results in integer valued solution. This is important as it avoids the need of discretizing solutions which can cause error and poor performance [8].

To facilitate optimization of Φ , we need to convert P into a vector. We define the vectorization of a matrix as the concatenation of its rows¹, denoted by $\text{vec}()$. Let $p \triangleq \text{vec}(P)$. To get the form of Φ in terms of vector p , new denotations are needed. Let

$$\begin{aligned} \text{vec}(X^\top P Y) &\triangleq Bp, \quad X^\top P 1_n \triangleq Cp, \quad Y^\top P^\top 1_m \triangleq Dp, \\ \tilde{x}^\top P 1_n &\triangleq a^\top p, \quad 1_m^\top P \tilde{Y} \triangleq b^\top p \end{aligned}$$

Based on the fact $\text{vec}(M_1 M_2 M_3) = (M_1 \otimes M_3^\top) \text{vec}(M_2)$ for any matrices M_1 , M_2 and M_3 , we have matrices $B = X^\top \otimes Y^\top$, $C = X^\top \otimes 1_n^\top$, $D = 1_m^\top \otimes Y^\top$ and vectors $a = \tilde{x} \otimes 1_n$, $b = 1_m \otimes \tilde{y}$. Here \otimes denotes the Kronecker product. With the above preparation, $\Phi(P)$ can be written in terms of vector p as

$$\begin{aligned} \Phi(p) = & (b - \mu 1_{mn})^\top p - \frac{1}{1_{mn}^\top p} \|Dp\|^2 + \min_{s,R} \{s^2 (a^\top p - \frac{1}{1_{mn}^\top p} \|Cp\|^2) \\ & - 2s \cdot \text{tr}(R[\text{mat}(Bp) - \frac{1}{1_{mn}^\top p} Cpp^\top D^\top])\} \end{aligned} \quad (6)$$

where $\text{mat}()$ denotes converting a vector into a matrix, which can be seen as inverse of the operator $\text{vec}()$.

¹This is different from the conventional definition.

To facilitate optimization of Φ , we need to get the formula of the gradient of Φ . As Φ involves minimization operations, it's difficult to directly derive the formula of $\frac{\partial \Phi}{\partial p}$. To address this issue, we appeal to the result of Danskin's theorem [9] (page 245 therein), which in our case states that if $\Phi(p, s, R)$ is concave in p for each s and R (this can be proved analogously as the proof of Proposition 1) and the feasible regions of s and R are compact, then $\Phi(p) = \min_{s, R} \Phi(p, s, R)$ has gradient:

$$\begin{aligned} \frac{\partial \Phi(p)}{\partial p} &= \frac{\partial \Phi(p, \hat{s}, \hat{r})}{\partial p} = b - \mu 1_{mn} - \frac{2}{1_{mn}^\top p} D^\top D p \\ &+ \frac{\|Dp\|^2}{(1_{mn}^\top p)^2} 1_{mn} + \hat{s}^2 (a - \frac{2}{1_{mn}^\top p} C^\top C p + \frac{1}{(1_{mn}^\top p)^2} \|Cp\|^2 1_{mn}) \\ &- 2\hat{s}\{B^\top \hat{r} - \frac{1}{1_{mn}^\top p} [D^\top (p^\top C^\top \otimes I_d) + C^\top (I_d \otimes p^\top D^\top)] \hat{r} \\ &+ \frac{1}{(1_{mn}^\top p)^2} \text{tr}(\hat{R} C p p^\top D^\top) 1_{mn}\} \end{aligned} \quad (7)$$

where \hat{s} and \hat{R} satisfy $\Phi(p, \hat{s}, \hat{R}) = \min_{s, R} \Phi(p, s, R)$. The optimal \hat{s} and \hat{R} can be obtained by the method described previously. Here the vector $\hat{r} \triangleq \text{vec}(\hat{R}^\top)$ and I_d denotes the $d \times d$ identity matrix.

5 PF based optimization

The PF algorithm [4] is used to optimize Φ by constructing an interpolation function between a convex function $\|p\|^2$ and the concave function Φ ,

$$E_\lambda = (1 - \lambda) \|p\|^2 + \lambda \Phi(p)$$

and gradually increasing λ from 0 to 1 so that E_λ gradually transitions from the convex function $\|p\|^2$ to the concave function Φ . With each value of λ , E_λ is locally minimized. We refer the reader to [5] for detail.

6 Experimental results

We compare our method with state-of-the-art methods including RPM-PF [5], RPM [1], Go-ICP [10], CPD [6] and gmmreg [11]. To ensure fairness, for RPM-PF, transformation is not regularized. We implement all the methods in MATLAB on a PC with a 3.3 GHz CPU and 16 G RAM. For methods only outputting point correspondence, affine transformation is used to warp the model point set. For our method, we set parameters $\underline{s} = 0.5$ and $\bar{s} = 1.5$.

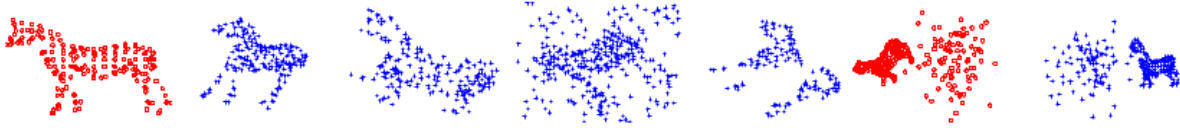


Figure 2: First 5 columns: model point set (left column) and examples of scene sets in the deformation, positional noise, outlier and occlusion tests, respectively (columns 2 to 5). Last 2 columns: examples of model (column 6) and scene (right column) point sets in the coexisting outlier test.

Following [12, 3], we test a method’s robustness to non-rigid deformation, positional noise, outliers, occlusion and coexisting outliers, as illustrated in the second to fifth column of Fig. 2. Also, to test a method’s ability to handle rotation and scale changes, random rotation with rotation angle less than 60 degree and random uniform scaling with scale factor within range $[0.5, 1.5]$ are applied to the prototype shape when generating the scene point set.

The average matching accuracies (fraction of correct matches) by different methods are presented in Fig. 3. One can see that our method performs considerably better than other methods. This demonstrates our method’s robustness to various types of disturbances. Examples of matching results by different methods in the coexisting outlier test are shown in Fig. 4. The average running times (in second) by different methods are 3.6054 for our method, 4.1525 for RPM-PF, 1.8160 for RPM, 5.7997 for Go-ICP, 0.0612 for CPD and 0.2865 for gmmreg. It’s clear our method is efficient.

7 Conclusion

We proposed a PF based point matching method in this letter by adapting the method of [5] to use the similarity transformation. Due to nonlinearity of 3D similarity transformation, this is a nontrivial extension of the method of [5]. Experimental results demonstrate better robustness of the proposed method over state-of-the-art methods.

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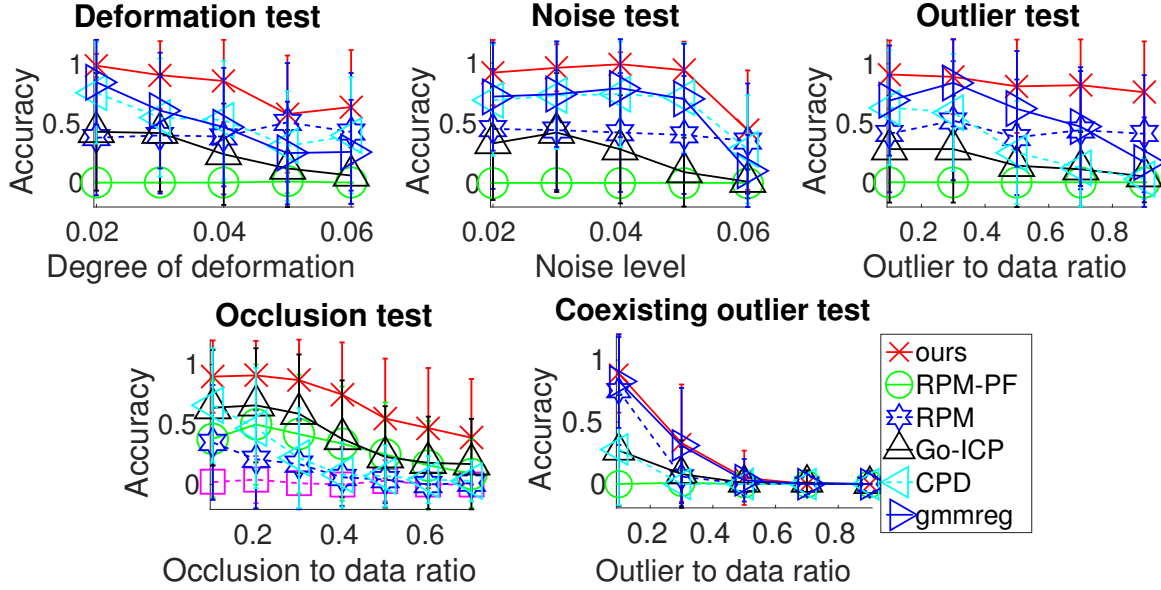


Figure 3: Average matching accuracies by different methods in the 5 categories of tests. The error bars indicate standard deviations of the methods over 100 random trials.

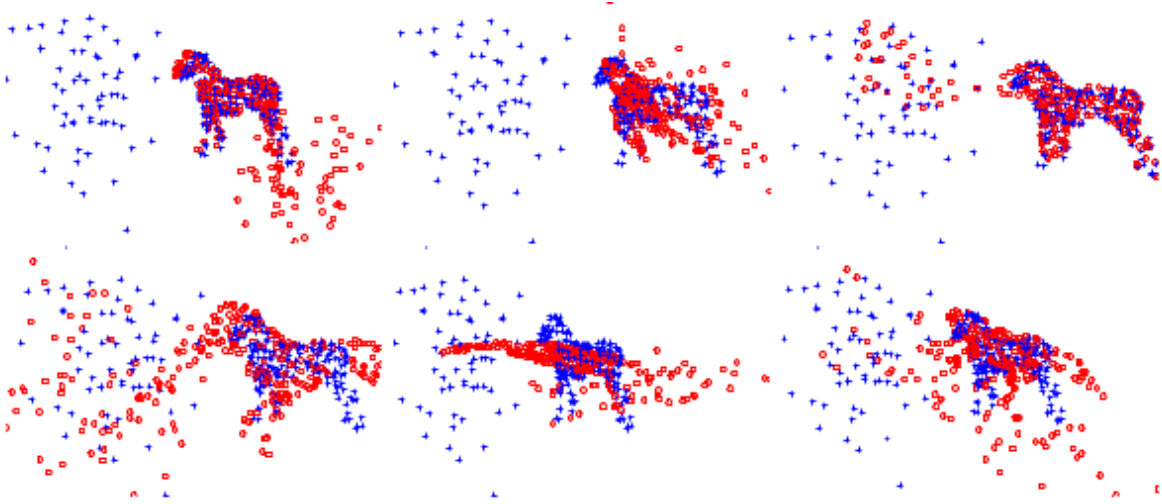


Figure 4: Examples of matching results by (from left to right and from top to bottom) our method, RPM-PF, RPM, Go-ICP, CPD and gmmreg in the coexisting outlier test.

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